AP CALCULUS BC

Unit 6 Outline – Integration Techniques

DATE	CONCEPT	IN-CLASS SAMPLE PROBLEMS
10/18	**INTEGRATION AND ANTIDERIVATIVES** **INITIAL VALUE PROBLEMS**	Find the general antiderivatives of each of the following using your knowledge of how to find derivatives. 1. $f'(x) = x^7 - 6x + 8$ 2. $\int\left(\frac{1}{5} - \frac{2}{x^3} + 2x\right)dx$ 3. $F'(x) = (x-2)^3$ Evaluate each indefinite integral 4. $\int x^{-1}dx$ 5. $\int \cos 3x\,dx$ 6. $\int e^{2x}dx$ 7. Find a function y that satisfies $\frac{dy}{dx} = x^2 + 2x + 5$ and $y\big\|_{x=1} = 3$ 8. Find the <u>particular solution</u> to the equation $\frac{dy}{dx} = e^x - 6x$ whose graph passes through the point $(1, 0)$. 9. Find the particular solution to the following differential equation $\frac{d^2y}{dx^2} = 12x^2 + 6x - 8$ and $y'(1) = 5$ and $y(1) = 6$.

AP MULTIPLE CHOICE

$$\int 5x\left(\sqrt{x} - x^2\right)dx =$$

(A) $\frac{15\sqrt{x}}{2} - 15x^2 + C$

(B) $5x - \frac{5x^4}{4} + C$

(C) $2x^{5/2} - \frac{5x^4}{4} + C$

(D) $\frac{25x^{5/2}}{2} - \frac{5x^4}{4} + C$

(E) $\frac{5x^{7/2}}{3} - \frac{5x^6}{6} + C$

If $f'(x) = \frac{2}{x}$ and $f(\sqrt{e}) = 5$, then $f(e) =$

(A) 2 (B) $\ln 25$ (C) $5 + \frac{2}{e} - \frac{2}{e^2}$ (D) 6 (E) 25

HOMEWORK	Worksheet 39

DATE	CONCEPT	IN-CLASS SAMPLE PROBLEMS
10/21	**ANTIDERIVATIVES** **"MOST COMPLICATED" RULE**	Ex. 1 What is $\frac{d}{dx}\left(f\left(g\left(x\right)\right)\right)$? Ex. 2 $\int \frac{2x+3}{\sqrt[3]{x^2+3x+6}}dx =$ Ex. 3 $\int \frac{\sin x}{\cos^2 x}dx =$ Ex. 4 $\int \sin^4\left(3x\right)\cos\left(3x\right)dx =$ Ex. 5 $\int \frac{x^3}{7-x^4}dx =$

AP MULTIPLE CHOICE

$$\int \frac{e^x}{1+e^x}\,dx =$$

(A) $\ln\left(\frac{1}{e^x}+1\right)+C$

(B) $\ln\left(1+e^x\right)+C$

(C) $x-\ln\left(1+e^x\right)+C$

(D) e^x+x+C

(E) $\tan^{-1}\left(e^x\right)+C$

HOMEWORK	Worksheet 40

DATE	CONCEPT	IN-CLASS SAMPLE PROBLEMS
10/22	**ANTIDERIVATIVES** **INTEGRATION BY SUBSTITUTION**	Ex. 1 $\int \frac{2x+3}{\sqrt[3]{x^2+3x+6}}dx =$ Ex. 2 $\int \frac{\sin x}{\cos^2 x}dx =$

AP MULTIPLE CHOICE

Using the substitution $u = x^2 - 3$, $\int_{-1}^{4} x\left(x^2-3\right)^5\,dx$ is equal to which of the following?

(A) $2\int_{-2}^{13} u^5\,du$

(B) $\int_{-2}^{13} u^5\,du$

(C) $\frac{1}{2}\int_{-2}^{13} u^5\,du$

(D) $\int_{-1}^{4} u^5\,du$

(E) $\frac{1}{2}\int_{-1}^{4} u^5\,du$

HOMEWORK	Worksheet 41

DATE	CONCEPT	IN-CLASS SAMPLE PROBLEMS
10/23	**THE DEFINITE INTEGRAL**	Evaluate the definite integral. 1. $\int_1^4 -x^{-2}dx$ 2. $\int_{\pi}^{2\pi} \sin x dx$ 3. $\int_1^e \frac{1}{x}dx$ 4. $\int_0^2 x\left(x^2-3\right)^3 dx$ 5. $\int_0^1 (4-2x)e^{8x-2x^2} dx$ 6. $\int_0^1 \frac{6x}{1+x^2} dx$

AP MULTIPLE CHOICE

If $\int_0^1 f(x)\,dx = 2$ and $\int_0^4 f(x)\,dx = -3$, then $\int_1^4 (3f(x)+2)\,dx =$

(A) -13 (B) -9 (C) -7 (D) 3 (E) 21

$\int_0^1 x\sqrt{1+8x^2}\,dx =$

(A) $\frac{1}{24}$ (B) $\frac{13}{12}$ (C) $\frac{9}{8}$ (D) $\frac{52}{3}$ (E) 18

HOMEWORK	Worksheet 42

DATE	CONCEPT	IN-CLASS SAMPLE PROBLEMS
10/24	**INTEGRATION BY PARTS**	**Warm-Up:** Evaluate $\frac{d}{dx}(uv)$, given that u and v are both functions of x. Evaluate the indefinite integral: **Ex. 1** $\int x\cos x dx =$ **Ex. 2** $\int 2xe^{4x}dx =$ **Ex. 3** $\int x\ln x dx =$ **Ex. 4** $\int \ln x dx =$ **Ex. 5** $\int \arctan x dx$

AP MULTIPLE CHOICE

. Let f be a differentiable function such that $\int f(x)\sin x\,dx = -f(x)\cos x + \int 4x^3\cos x\,dx$. Which of the following could be $f(x)$?

(A) $\cos x$ (B) $\sin x$ (C) $4x^3$ (D) $-x^4$ (E) x^4

HOMEWORK	Worksheet 43

DATE	CONCEPT	IN-CLASS SAMPLE PROBLEMS
10/25	**INTEGRATION BY PARTS**	Evaluate the definite integral: **Ex. 1** $\int_0^{\pi/2} \theta^2 \sin 2\theta \, d\theta$ **Ex. 2** $\int_0^{\pi/2} x^3 \cos 2x \, dx$

AP MULTIPLE CHOICE

x	2	4
$f(x)$	7	13
$g(x)$	2	9
$g'(x)$	1	7
$g''(x)$	5	8

The table above gives selected values of twice-differentiable functions f and g, as well as the first two derivatives of g. If $f'(x) = 3$ for all values of x, what is the value of $\int_2^4 f(x)g''(x)\,dx$?

(A) 63 (B) 69 (C) 78 (D) 84 (E) 103

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x	$f(x)$	$f'(x)$
0	2	5
4	–3	11

The function f has a continuous derivative. The table above gives values of f and its derivative for $x = 0$ and $x = 4$. If $\int_0^4 f(x)\,dx = 8$, what is the value of $\int_0^4 xf'(x)\,dx$?

(A) –20 (B) –13 (C) –12 (D) –7 (E) 36

HOMEWORK	Worksheet 44

DATE	CONCEPT	IN-CLASS SAMPLE PROBLEMS
10/28	**QUIZ**	Initial Value Problems, Most Complicated Rule, Substitution, Definite Integrals
HOMEWORK		None

DATE	CONCEPT	IN-CLASS SAMPLE PROBLEMS
10/29	**PARTIAL FRACTION DECOMPOSITION**	**Ex. 1** Express as a sum of partial fractions: $\frac{5x-3}{x^2-2x-3}$ Evaluate the integral: **Ex. 2** $\int \frac{3x+11}{x^2-x-6}dx$ **Ex. 3** $\int \frac{x+5}{x^2+x-2}dx$ *** Ex. 4** $\int \frac{x-13}{2x^2-7x+3}dx$

AP MULTIPLE CHOICE

$\int_0^1 \frac{5x+8}{x^2+3x+2}dx$ is

(A) $\ln(8)$ (B) $\ln\left(\frac{27}{2}\right)$ (C) $\ln(18)$ (D) $\ln(288)$ (E) divergent

HOMEWORK	Worksheet 45

DATE	CONCEPT	IN-CLASS SAMPLE PROBLEMS
10/30	**IMPROPER INTEGRALS**	**Evaluate each integral:** 1) $\int_0^1 \frac{dx}{\sqrt{x}}$ 2) $\int_{-1}^1 \frac{dx}{x^{2/3}}$ 3) $\int_1^\infty \frac{dx}{x}$ 4) $\int_1^\infty \frac{dx}{x^2}$ 5) $\int_{-\infty}^0 \frac{dx}{\sqrt{3-x}}$ 6) $\int_{-2}^1 \frac{1}{x^2}dx$

AP MULTIPLE CHOICE

$\int_1^\infty \frac{x^2}{\left(x^3+2\right)^2}dx$ is

(A) $-\frac{1}{9}$ (B) $\frac{1}{9}$ (C) $\frac{1}{3}$ (D) 1 (E) divergent

HOMEWORK	Worksheet 46

DATE	CONCEPT	IN-CLASS SAMPLE PROBLEMS
10/31	**INTEGRATION USING DIVISION FIRST** **REVIEW**	**Ex. 1** $\int \frac{x^2-3x+5}{x-4}dx$ **Ex. 2** $\int \frac{x^2-3x+7}{x+5}dx$

AP MULTIPLE CHOICE

$$\int \frac{x^3+5}{x^2}dx =$$

(A) $1-\frac{10}{x^3}+C$

(B) $\frac{3x}{4}+\frac{15}{x^2}+C$

(C) $\frac{x^2}{2}-\frac{5}{x}+C$

(D) $\frac{x^2}{2}-\frac{5}{3x^3}+C$

(E) $-\frac{x^3}{4}-5+C$

HOMEWORK	Worksheet 47

DATE	CONCEPT	IN-CLASS SAMPLE PROBLEMS
11/4	**INTEGRATION REVIEW**	
HOMEWORK		Worksheet 48

DATE	CONCEPT	IN-CLASS SAMPLE PROBLEMS
11/5	**INTEGRATION EXAM**	**Good luck on today's exam!**
HOMEWORK		None